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GAMD-3388
Report Number

United States Atomic Energy Commission
Division of Technical Information

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SOME FUNDAMENTAL PROPERTIES OF SHOCK WAVES

Nuclear/Chemical Pulse Reaction Propulsion Project

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Air Force Special Weapons Center
Contract AF29(601)-2807

June 1, 1961
Project 52

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SOME FUNDAMENTAL PROPERTIES OF SHOCK WAVES

~~The purpose of this analysis is to examine generally~~ the Rankine-Hugoniot conditions, and then ~~to apply them~~ to develop the similarity solution for a strong spherical shock. Several interesting theorems can be observed about the jump conditions. The similarity solution has been considered in the literature, but it apparently has not been carried out completely; it turns out that a simple solution is available to this important problem.

The Fluid Dynamics Equations

In integral form the fluid dynamics equations for conservation of mass, momentum, and energy are

$$\frac{\partial}{\partial t} \int dv\rho + \int d\underline{A} \cdot \left[\rho\underline{u}\right] = 0$$

$$\frac{\partial}{\partial t} \int dv\rho\underline{U} + \int d\underline{A} \cdot \left[\rho\underline{u}\,\underline{U} + \underline{P}\right] = 0$$

$$\frac{\partial}{\partial t} \int dv\rho(E + \tfrac{1}{2}U^2) + \int d\underline{A} \cdot \left[\rho\underline{u}(E + \tfrac{1}{2}U^2) + \underline{P}\cdot\underline{U}\right] = 0$$

The volume integrals are the mass, momentum, and total energy enclosed in the volume selected. $\underline{U}$ is the velocity in a fixed coordinate system and $\underline{u}$ is the velocity in any moving system; the pressure tensor is $P = pI$, and p is the pressure, I the unit tensor; E is the internal energy per unit mass.

To get the jump conditions the volume is taken to include the shock. Furthermore, great simplifications are possible. First of all, if the volume element is taken small enough all the time derivative terms drop out.

Also, if the volume element is made thin enough, the surface integrals become essentially one dimensional, and

$$\left[\rho u\right] = 0$$
$$\left[\rho u U + p\right] = 0$$
$$\left[\rho u(E + \tfrac{1}{2} U^2) + pU\right] = 0$$

The brackets indicate differences between the enclosed quantities on each side of the shock. Throughout this analysis, only normal shocks are considered; for oblique shocks, cosine factors enter the above relations. Of course, the result of the above demonstration is that the Rankine-Hugoniot conditions based on steady, one-dimensional flow are completely satisfactory, and hold for non-steady, curved shocks.

The Shock Jump Conditions

For a shock of speed S advancing into a stationary medium, p_o, ρ_o, E_o, $U_o = 0$, then $u = U - S$ and the quantities just behind the shock p_1, ρ_1, E_1, U_1 are

$$\rho_o(U_1 - S) = -\rho_o S$$

$$\rho_o(U_1 - S)U_1 + p_1 = p_o$$

$$\rho_1(U_1 - S)(E_1 + \tfrac{1}{2} U_1^2) + p_1 U_1 = -\rho_o S E_o$$

There are several methods of rating shock strength; an effective one is to rate the strength of a shock by its compression ratio $k = \rho_o / \rho_1$. As is known and will be shown later, the compression ratio for a strong shock in a diatomic gas is 1:6. In terms of the compression ratio the properties behind the shock can be determined immediately.

$$U_1 = (1 - k)S$$
$$p_1 = p_o + (1 - k)\ \rho_o S^2$$
$$E_1 = E_o + (1 - k)\frac{p_o}{\rho_o} + \frac{1}{2}(1 - k)^2 S^2$$

In this form the results are general and do not depend on the equation of state.

For a strong shock, simply,

$$E_1 = \frac{1}{2} U_1^2$$

This proves the

<u>Theorem</u>: The kinetic and internal energy of a strong shock are equipartitioned.

To go further with the jump conditions, the gas law is used.

$$\frac{p_1}{\rho_1} = (\gamma - 1)E_1 = k\frac{p_o}{\rho_o} + k(1 - k)S^2$$

Now all the quantities p_1, E_1, U_1, S are available in terms of the compression ratio.

$$\frac{p_1}{p_o} = \frac{\frac{\gamma + 1}{\gamma - 1} - k}{\frac{\gamma + 1}{\gamma - 1}k - 1}$$

$$\frac{E_1}{E_o} = 1 + (\gamma - 1)(1 - k)\left[1 + \frac{\gamma(1 - k)}{(\gamma + 1)k - (\gamma - 1)}\right]$$

$$\frac{U_1}{S} = 1 - k$$

$$\frac{c^2}{S^2} = \frac{1}{2}(\gamma + 1)k - \frac{1}{2}(\gamma - 1)$$

And c is the sound speed $c^2 = \gamma(\gamma - 1)E_o$. These results for $\gamma = 1.4$ are shown in Fig. 1. Also given are the kinetic energy fraction and a measure of the entropy change $\mathcal{J}$, the factor $f = e^{-\mathcal{J}}$, such that

$$f\frac{p}{p_o} = \left(\frac{\rho}{\rho_o}\right)^{\gamma}$$

It is possible to make several interesting observations from the figure, particularly in the acoustic limit and the strong shock limit. It is apparent that

Theorem: The kinetic energy of any shock cannot exceed its interval energy.

By observing that the curves for p_o/p_1 and c^2/S^2 are close for any compression ratio,

$$p_1 c^2 \approx p_o S^2$$

This means that it is essentially equivalent to rate shocks of all strengths by their overpressure p_1 or their speed squared S^2.

The Strong Spherical Shock

For the case of a strong shock produced by a spherical charge of yield H_o, a similarity solution can be attempted for the profiles at position r behind the spherical shock at R, in terms of the dimensionless variable $\xi = r/R$.

$$\frac{p}{p_1} = \tilde{p}(\xi)$$

$$\frac{\rho}{\rho_1} = \tilde{\rho}(\xi)$$

$$\frac{U}{U_1} = \tilde{U}(\xi)$$

The conservation of energy is

$$H_o = \int_0^R 4\pi r^2 dr\, \rho(E + \tfrac{1}{2}U^2) = \int_0^R 4\pi r^2 dr\, (\tfrac{5}{2}p + \tfrac{1}{2}\rho U^2)$$

And the strong shock conditions are

$$p_1 = \tfrac{5}{6}\rho_o S^2 \qquad \rho_1 = 6\rho_o \qquad U_1 = \tfrac{5}{6}S$$

so that

$$H_o = CMU_1^2$$

where

$$M = \tfrac{4}{3}\pi\rho_o R^3$$

$$\tfrac{1}{3}C = \int_0^1 3\xi^2\, d\xi(\tilde{p} + \tilde{\rho}\,\tilde{U}^2)$$

If a similarity solution is possible, then C is indeed a constant and the equation for the motion of the strong spherical shock is

$$R^3\dot{R}^2 = H_o/\tfrac{4}{3}\pi\rho_o(\tfrac{5}{6})^2 C$$

and its solution is

$$R^{5/2} = 3\sqrt{H_o/\tfrac{4}{3}\pi\rho_o C}\quad t$$

The parameter C must be found from the $\tilde{p}$, $\tilde{\rho}$, $\tilde{U}$ profiles. The momentum equation provides

$$\frac{\partial p}{\partial r} = -\rho\frac{d}{dt}U$$

so that the pressure profile is determined if $\tilde{\rho}$, $\tilde{U}$ profiles can be found to satisfy the continuity equation.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\,\rho U r^2 = 0$$

Since $\tilde{\rho}$, $\tilde{U}$ range between 0 and 1, it is sufficient to use a simple power law with unknown exponents for each.

$$\tilde{\rho} = \xi^m$$
$$\tilde{U} = \xi^n$$

Then the continuity condition reduces to

$$m\xi = (m + n + 2)\xi^{n-1} U_1$$

so that $n = 1$ and $m = 3(k^{-1} - 1) = 15$. The total mass is

$$M = \int_0^R 4\pi r^2 dr\,\rho_1 \xi^m = 4\pi \rho_1 R^3 \int_0^1 \xi^{m+2}\, d\xi = \frac{4}{3}\pi \rho_o R^3$$

The success of the similarity solution is now assured, and the pressure profile must fall out of the momentum equation.

$$R\frac{\partial p}{\partial r} = \rho_1(\xi U_1 - \xi U^1 R - U_1^2)\xi^{m+1}$$

$$= 10\, p_1 \xi^{m+1}$$

Thus

$$p - p_c = \frac{10}{17} p_1 \xi^{17}$$

and p_c is the pressure at the center which is determined from the condition that $p = p_1$ at $\xi = 1$, so that $p_c = \frac{7}{17} p_1$.

In summary, the profiles behind a strong spherical shock are

$$\tilde{U} = \xi$$
$$\tilde{\rho} = \xi^{15}$$
$$\tilde{p} = \frac{7}{17} + \frac{10}{17}\xi^{17}$$

and these are shown in Fig. 2.

Finally, the constant C is

$$C = 3\int_0^1 3\xi^2 d\xi\,(\tilde{p} + \tilde{\rho}\,\tilde{U}^2)$$

$$= \frac{39}{20} = 1.95$$

$$H_o = 1.95\, MU_1^2$$

With this value for C the motion of the strong spherical shock is completely determined.

$$R^5 = 1.10\,\frac{H_o}{\rho_o} t^2$$

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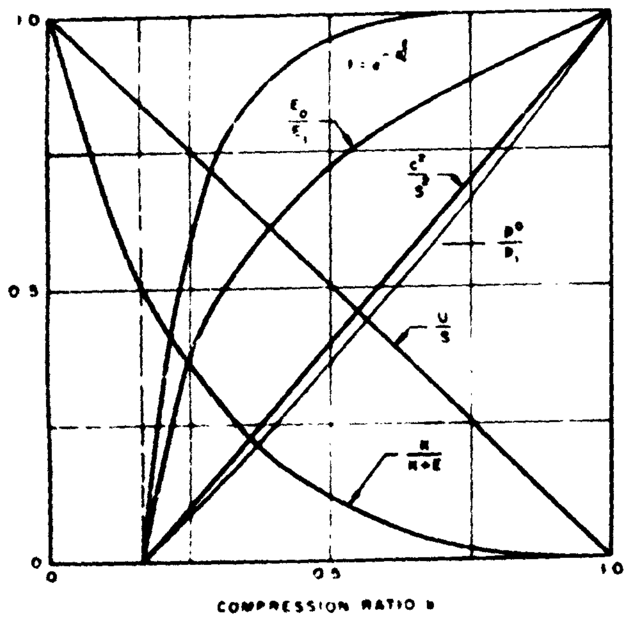

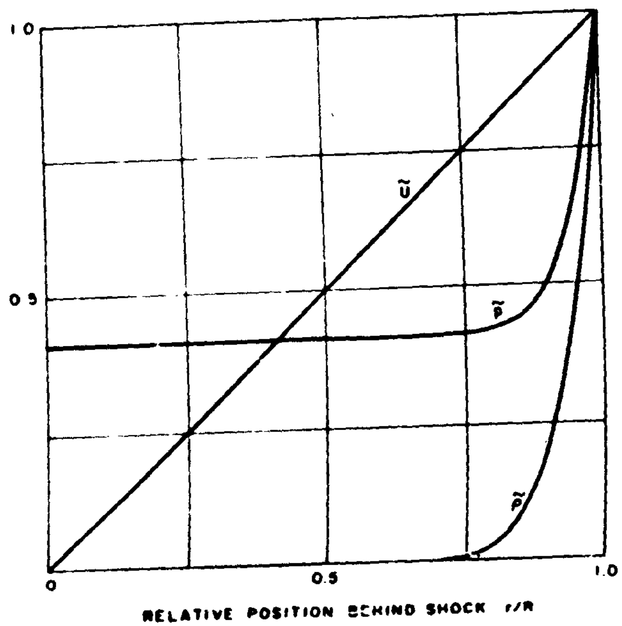

Pressure, Density, and Velocity
Behind Strong Spherical Shock